Candidate's Number .... Math's teacher ....

SYDNEY TECHNICAL HIGH SCHOOL
(Est. 1911)

2000 TRIAL HIGHER SCHOOL CERTIFICATE

MATHEMATICS

2 UNIT

Time Allowed - Three hours
(plus 5 minutes reading time)

DIRECTION TO CANDIDATES:

- All questions may be attempted
- All questions are of equal value.

  Approximate marks are shown.
- All working should be shown in every question.
- Full marks may not be awarded for careless of badly arranged work
- Standard Integrals are printed on the last page of this examination.
- Approved calculators may be used.
- Each question attempted is to be started ON A NEW PAGE, clearly marked with the number of the question and your name and class on the top right hand side of the page.

  This question paper must be handed in with your answers at the conclusion of the. examination

QUESTION	Mark
Question 1	
Question 2	
Question 3	
Question 4	
Question 5	
Question 6	
Question 7	
Question 8	
Question 9	
Question 10	
TOTAL	

QUESTION 1

1 a) Factorise $4x^2 - 25$

1 b) Find $\log_3 27$

2 c) Find sin 1.3 radians correct to 3 decimal places.

2 d) Simplify $\frac{x}{3} - \frac{3x-1}{10}$

2 e) If $\sqrt{45} + \sqrt{A} = 5\sqrt{5}$ find A.

2 f) Solve $4 - 3x < 9$

2 g) For the parabola $x^2 = -8(y+2)$ find

i) co-ordinates of the vertex.

ii) focal length

QUESTION 2

6 a) Differentiate the following

i) $(x^2 - 6)^4$

ii) $x \tan x$

iii) $\sqrt[3]{x}$

3 b) Evaluate $\int_1^6 \frac{x-1}{x} dx$ (exact value)

3 c) i) Write down the discriminant of $2x^2 - kx + k$

ii) For what values of k does $2x^2 - kx + k$ have real roots ?

QUESTION 3

8 a) A , B and C are the points (1,0), (3,2) and (-1,8) respectively

i) Draw a diagram to show the points A , B and C.

ii) Find the length BC.

iii) Show that the equation of the line BC is $3x+2y-13=0$

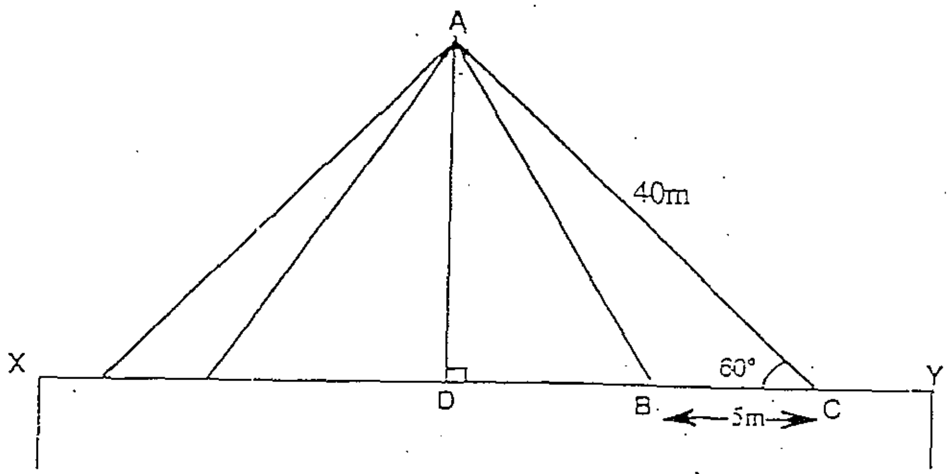
iv) Find the perpendicular distance from A to the line BC.

v) Find the area of the triangle ABC.

vi) If ABCD is a parallelogram write down the co-ordinates of D.

vii) Find the co-ordinates of the point where the diagonals AC and BD meet.

4 b)



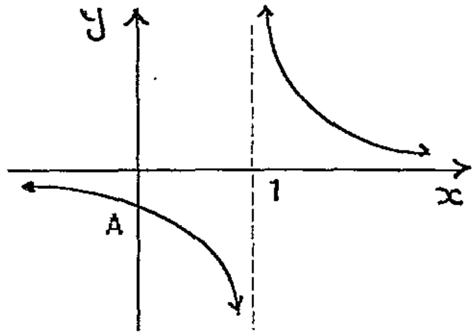
A horizontal bridge joins points X and Y. Cables were used to support the bridge as shown in the diagram. The distance between the cables AB and AC was 5 metres while AC was 40 metres and $\angle ACB = 60°$

i) Find the exact height of A above the bridge.

ii) Use the cosine rule to find the length of the cable AB.

QUESTION 4

7 a)



The diagram shows the graph of $y = \dfrac{1}{x-1}$

i) The curve cuts the y-axis at A. Find the co-ordinates of A.

ii) Show that the gradient of the normal at A is 1

iii) Find the equation of the normal at A.

iv) The normal cuts the curve again at B. Find the co-ordinates of B.

5 b) A tower is made by adding layers of cubes.Each layer is a square
The number of cubes in the side of each square layer forms an Arithmetic Progression.The first 3 terms are 80, 76, 72......

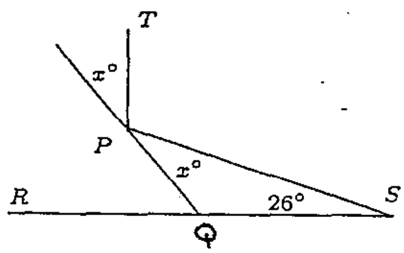
i) Find the eighteenth term of the sequence.

ii) Find the sum of the first eighteen terms.

iii) If 685,580 cubes are available,is it possible to build this eighteen layer tower?

QUESTION 5

2



a) In this diagram PT⊥RS . Copy the diagram onto your page.
Find the value of x, giving reasons.

3 b) Find the volume when the curve $y = e^{3x}$ between $x= 0$ and $x= 2$ is rotated about the x axis.

3 c) A bottle of cleaner fluid has a spray nozzle which is defective. When used it only delivers 90% of its previous spray.On its first spray it delivers 15 mL and the user continues to spray.

i) On the fifth spray how many mL's does it deliver?

ii) What is the total limiting amount the spray can deliver.

4 d) Given $\log 6 = x$ and $\log 2 = y$ .Find in terms of x and y

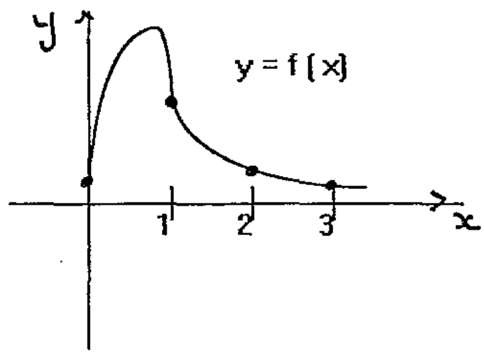
i) $\log 3$

ii) $\log 24$

iii) Evaluate $\dfrac{2x}{y}$ to 2 decimal places.

QUESTION 6

5 a)



The table shows the values of $f(x)$ for 4 values of x for the graph $y=f(x)$

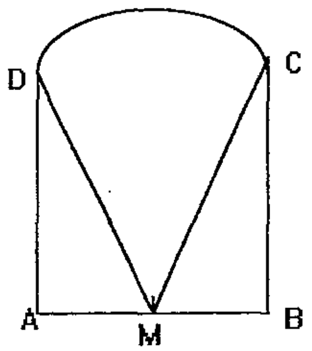
x	0	1	2	3
$f(x)$	1	15	5	2

i) Use the Trapezoidal Rule and the 4 function values to evaluate

$$\int_0^3 f(x)\,dx$$

ii) From the diagram, decide if this approximation is an over-estimate or an under-estimate of the true value of the integral.Give a brief reason.

7 b)



This is a glaziers plan for a window.DC is an arc of a circle centre M with radius 120 cm.ABCD is a rectangle where AM=MB=60 cm

i) Find the size of $\angle DMA$

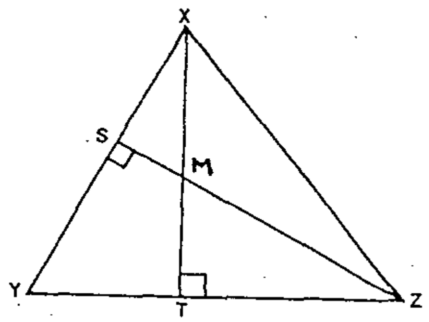
ii) Find the size of $\angle DMC$

iii) Find the length of arc DC.

iv) Find the total area of glass needed for the window.

QUESTION 7

6 a)



In the diagram $ZS \perp XY$ and $XT \perp YZ$. Redraw the diagram on your paper.

i) Explain why $\angle YXT = \angle SZY$

ii) Prove ΔTXY is similar to ΔTZM

iii) If $XT = 8$ cm, $YT = 5$cm and SZ bisects XT. Find the length of TZ.

6 b) The volume of water in a tank with capacity 400 litres is controlled by two pumps. When pump A operates the volume of water in the tank is given by $V = 400 - 4t - \dfrac{t^2}{5}$ (t is in minutes)

i) Find the volume after 20 minutes.

ii) Find the rate of flow of water after 20 minutes. Explain your answer.

At this time (20 minutes) pump A stops and pump B begins. Pump B fills the tank at a rate given by $\dfrac{dV}{dt} = \dfrac{t}{10} + 5$

iii) How much water will be in the tank 10 minutes after pump B starts.

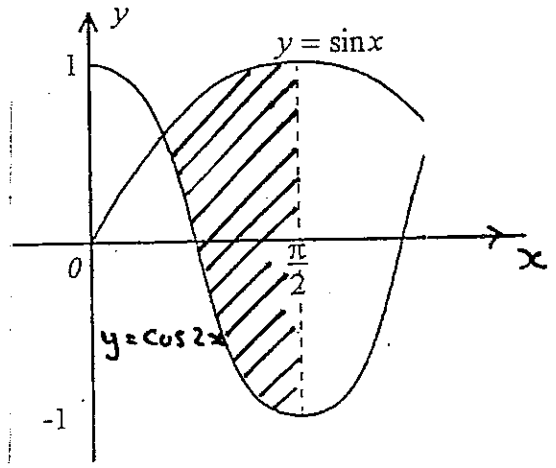
QUESTION 8

3 a) The population of insects is given by $P = 6000e^{0.3t}$ where t is in days

i) Find the population when t=5.

ii) How long does it take for the population to reach 50000.

5 b)



The diagram shows parts of the curves *y=sinx* and *y=cos2x*

i) Show that the curves intersect at $x = \frac{\pi}{6}$

ii) Calculate the shaded area. Leave the answer in exact form.

4 c) A bus company has established that the cost of a 1000 km journey is $C = 0.04v^2 + \frac{17500}{v} + 275$ dollars, where v is the average speed in kilometres per hour. At what speed should the buses travel to achieve a minimum cost.

QUESTION 9

2 a) The weight w, in kilograms, of a new baby in the first few weeks is given in the following table.

Time(t)	1st week	2nd	3rd	4th	5th	6th
Weight (w)	0.6	0.8	1.05	1.35	1.8	2.5

Use the information to find the signs of

i) $\frac{dw}{dt}$

ii) $\frac{d^2w}{dt^2}$

10 b) For the curve $y = x^3 - 3x^2$

i) Find the co-ordinates of any stationary points and determine their nature.

ii) Find the co-ordinates of any points of inflexion.

iii) Draw a neat sketch of the curve indicating stationary points, points of inflexion and intercepts.

Use your graph or otherwise to state the values of x for which the curve is

iv) increasing

v) concave down

vi) The curve and the line $y = -kx + 3k$ intersect. For what values of k will there be only one point of intersection.

QUESTION 10

3 a) If $\int_{p}^{p+1} 3x^2 dx = 19$ Find the values of p

9 b) A fisherman moors his boat in a river which is heavily effected by the ocean tides. To reach the open sea from his mooring he must cross a sandbar and his boat requires water at least 2 metres deep On a particular day high tide occurred at midnight and the depth (D) of water over the sandbar on the ensuing day is given by the formula $D = 2\cos\frac{4\pi}{25}t + 1$ where t is the number of hours after midnight.

i) What is the depth of the water at high tide

ii) What is the depth of the water at low tide.Explain your answer.

iii) What time elapses between high tides.

iv) Use the above information to draw the graph of $D = 2\cos\frac{4\pi}{25}t + 1$ for $0 \le t \le 24$.

v) Draw another line on your graph and using the two graphs indicate the times of the day (on your graph) when he would not be able to cross the sandbar in his boat

STANDARD INTEGRALS

$$\int x^n \, dx = \frac{1}{n+1} x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x} \, dx = \ln x, \quad x > 0$$

$$\int e^{ax} \, dx = \frac{1}{a} e^{ax}, \quad a \neq 0$$

$$\int \cos ax \, dx = \frac{1}{a} \sin ax, \quad a \neq 0$$

$$\int \sin ax \, dx = -\frac{1}{a} \cos ax, \quad a \neq 0$$

$$\int \sec^2 ax \, dx = \frac{1}{a} \tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax \, dx = \frac{1}{a} \sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2 + x^2} \, dx = \frac{1}{a} \tan^{-1} \frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} \, dx = \sin^{-1} \frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} \, dx = \ln\left(x + \sqrt{x^2 - a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} \, dx = \ln\left(x + \sqrt{x^2 + a^2}\right)$$

NOTE: $\ln x = \log_e x, \quad x > 0$

Sydney Tech 2000

Question 1

) $(2x-5)(2x+5)$

) 3

) $0.96355\ldots$

$= 0.964$

) $\frac{x}{3} - \frac{3x-1}{10}$

$= \frac{30x - 9x + 3}{30}$

$= \frac{x+3}{30}$

) $\sqrt{A} = 5\sqrt{5} - \sqrt{45}$

$= 5\sqrt{5} - 3\sqrt{5}$

$= 2\sqrt{5}$

$\therefore A = 20$

f) $4 - 3x < 9$

$-3x < 5$

$x > -5/3$

g) i) $(0,-2)$

ii) 2

Question 2

) i) $\frac{d}{dx}(x^2-6)^4 = 4 \cdot 2x \cdot (x^2-6)^3$

$= 8x(x^2-6)^3$

ii) $\frac{d}{dx} x\tan x = \tan x \cdot 1 + x \cdot \sec^2 x$

$= \tan x + x\sec^2 x$

iii) $\frac{d}{dx} x^{1/3} = 1/3\, x^{-2/3}$

) $\int_1^6 \frac{x-1}{x}\,dx = \int_1^6 1 - \frac{1}{x}\,dx$

$= [x - \ln x]_1^6$

$= 6 - \ln 6 - 1$

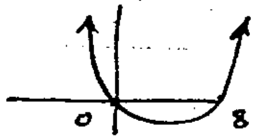
$= 5 - \ln 6$

c) i) $\Delta = k^2 - 4 \cdot 2 \cdot k$

$= k^2 - 8k$

ii) $\Delta \geq 0$

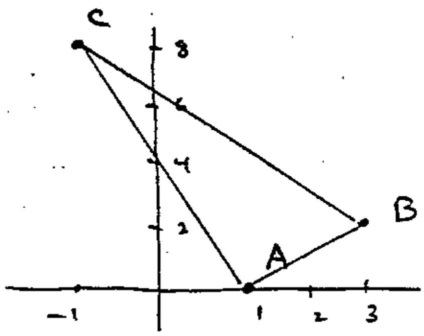
$\therefore k^2 - 8k \geq 0$



$k \leq 0,\ k \geq 8$

Question 3

a) i)



ii) $BC = \sqrt{6^2 + (-4)^2}$

$= \sqrt{52}$

iii) $mBC = \frac{8-2}{-1-3}$

$= -\frac{3}{2}$

$\therefore y - 2 = -3/2(x-3)$

$2y - 4 = -3x + 9$

$3x + 2y - 13 = 0$

iv) $d = \frac{|3\times 1 + 2\times 0 - 13|}{\sqrt{3^2+2^2}}$

$= \frac{10}{\sqrt{13}}$

v) $\text{Area} = \frac{1}{2} \times \sqrt{52} \times \frac{10}{\sqrt{13}}$

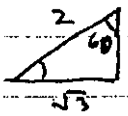
$= 10u^2$

vi) $D = (-3, 6)$

vii) diagonals meet a mid point of diagonal

$\therefore \left(\frac{1-1}{2}, \frac{0+8}{2}\right) = (0,4)$

3) i) $\sin 60^\circ = \frac{AD}{40}$



$AD = 40 \sin 60^\circ$

$= 40 \cdot \frac{\sqrt{3}}{2}$

$= 20\sqrt{3}$ metres

ii) $AB^2 = 40^2 + 5^2 - 2\times 40 \times 5 \times \cos 60^\circ$

$= 37.75$ metres

Question 4

a) i) $x = 0 \qquad y = \frac{1}{-1}$

$\therefore A = (0, -1)$

ii) $y = \frac{1}{x-1} = (x-1)^{-1}$

$\frac{dy}{dx} = -1 \cdot (x-1)^{-2}$

at $(0,-1)$

$\frac{dy}{dx} = -1$

$\therefore m_{\perp} = 1$

iii) $y + 1 = 1(x - 0)$

$y = x - 1$

iv) $\frac{1}{x-1} = x - 1$

$\therefore (x-1)^2 = 1$

$x - 1 = \pm 1$

$x = 1 \pm 1$

$\therefore x = 2$ or 0

$\therefore B\ (2, 1)$

b) i) $T_{18} = 80 + 17 \times (-4)$

$= 12$

ii) $S_{18} = 9[2\times 80 + 17 \times -4]$

$= 828$

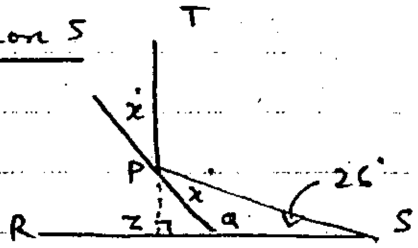
iii) $80^2 + 76^2 + \dots + 12^2 < 828^2$

$\therefore \qquad < 685584$

$\therefore$ 4[illegible]

Question 5

a)



Extend TP to RS

$\angle ZPQ = x^\circ$ (Vertically opposite

$\therefore 2x + 90 + 26^\circ = 180^\circ$ (angle sum of $\triangle$

$\therefore x = 32^\circ$

b) $V = \pi \int_0^2 (e^{3x})^2\, dx.$

$= \pi \int_0^2 e^{6x}\, dx$

$= \pi \left[\frac{e^{6x}}{6} \right]_0^2$

$= \pi \left[\frac{e^{12}}{6} - \frac{1}{6} \right]$

$= (e^{12} - 1)\frac{\pi}{6}$ or equivalent.

c) i) $T_5 = 15 \times (0.9)^4$

$= 9.8415$ ml

ii) limiting sum $= \frac{15}{1 - 0.9}$

$= 150$ ml

d) i) $\log 3 = \log 6 - \log 2$

$= x - y$

ii) $\log 24 = \log 6 + \log 4$

$= \log 6 + 2\log 2$

$= x + 2y$

iii) $\frac{2x}{y} = \frac{2\log 6}{\log 2}$

$= \frac{\log 36}{\log 2}$

by change of base rule

$= 5.17$ (2dec)

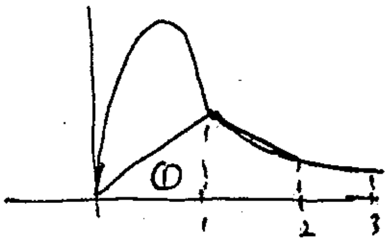
Question 6

a) i)

x	$f(x)$	w	$w \cdot f(x)$
0	1	1	1
1	15	2	30
2	5	2	10
3	2	1	2
		6	43

$$\therefore \int_0^3 f(x)\,dx = \frac{3-0}{6} \times 43$$
$$= 21.5$$

)



Underestimate. The first trapezium ① area is much less than the actual area.

i)

$\cos\theta = \frac{1}{2}$

$\theta = 60^\circ$

$\therefore \angle DMA = 60^\circ$

ii) $\angle DMC = 60^\circ$

iii) $\text{Arc} = 120 \times \frac{\pi}{3}$

$= 40\pi \quad (125.7 \text{ cm})$

iv) ~~Area = Now~~

$\tan 60^\circ = \frac{AD}{60}$

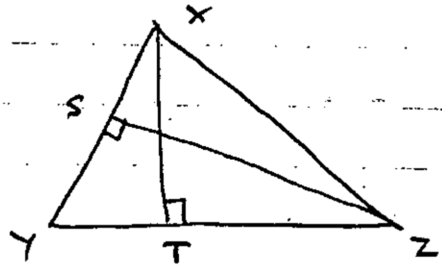
$AD = 60 \tan 60^\circ$

$\text{Area} = 60 \times 60 \tan 60^\circ + \frac{1}{2} \times 120^2 \times \frac{\pi}{3}$

$= 3600\sqrt{3} + 2400\pi$

$= 13775 \text{ cm}^2$

Question 7



i) $\angle YXT = 90^\circ - \angle Y$

$\angle YZS = 90^\circ - \angle Y$

$\therefore \angle YXT = \angle YZS$

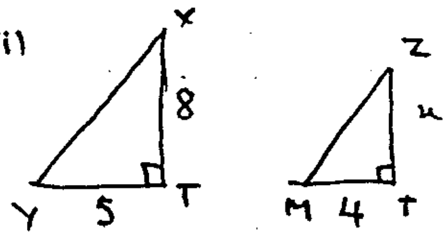
ii) In $\triangle TXY$ and $\triangle TZM$

$\angle YXT = \angle MZT$ (proven in (i))

$\angle YTX = \angle ZTM$ (right angle)

$\therefore \triangle TXY ||| \triangle TZM$ (equiangular)

iii)



Since triangles are similar

$\frac{x}{8} = \frac{4}{5}$

$5x = 32$

$x = 6.4 \text{ cm}$

b) i) $t = 20$

$V = 400 - 4\times 20 - \frac{20^2}{5}$

$= 240\ l.$

ii) $\frac{dV}{dt} = -4 - \frac{2t}{5}$

$t = 20$

$\frac{dV}{dt} = -12\ l/min$

flowing out of tank

iii)

$\frac{dV}{dt} = \frac{t}{10} + 5$

$V = \frac{t^2}{20} + 5t + c$

$t = 0$, $V = 240$ $\therefore c = 240$

$\therefore V = \frac{t^2}{20} + 5t + 240$

$t = 10$

$V = \frac{10^2}{20} + 5\times 10 + 240$

$= 295$ litres.

Question 8

a) i) $P = 6000e^{1.5}$

$= 2689$

ii) $50\,000 = 6000\,e^{0.3t}$

$\frac{50}{6} = e^{0.3t}$

$\ln\left(\frac{50}{6}\right) = 0.3t$

$t = 7.07$ days

b) i) $x = \pi/6$ $\sin \pi/6 = 0.5$

$\cos 2\pi/6 = 0.5$

$\therefore$ intersect at $x = \pi/6$

ii) Area $= \int_{\pi/6}^{\pi/2} \sin x - \cos 2x \, dx$

$= \left[-\cos x - \tfrac{1}{2}\sin 2x\right]_{\pi/6}^{\pi/2}$

$= \left[\cos x + \tfrac{1}{2}\sin 2x\right]_{\pi/2}^{\pi/6}$

$= \frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{4} - [0]$

$= \frac{3\sqrt{3}}{4} = 1.3\,u^2$

c) $C = 0.04v^2 + \frac{17500}{v} + 275$

$\frac{dC}{dv} = 0.08v - \frac{17500}{v^2}$

For minimum $\frac{dC}{dv} = 0$

$\therefore 0.08v - \frac{17500}{v^2} = 0$

$0.08v^3 = 17500$

$v^3 = \frac{17500}{0.08}$

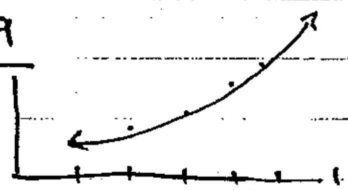
$V = 60.$[illegible]

v	50	60	80
$\frac{dC}{dv}$	−	0	+

$\therefore$ minimum cost when speed is 60 km/hr.

Question 9

a)



i) $\frac{dw}{dt} > 0$

ii) $\frac{d^2w}{dt^2} > 0$

b) i) $\frac{dy}{dx} = 3x^2 - 6x$

For stat pt $\frac{dy}{dx} = 0$

$\therefore 3x^2 - 6x = 0$

$3x(x-2) = 0$

$x = 0$ or 2

x	-1	0	1	2	3
$\frac{dy}{dx}$	+	0	−	0	+

$\therefore$ maximum at $(0,0)$

minimum at $(2,-4)$

ii) $\frac{d^2y}{dx^2} = 6x - 6$

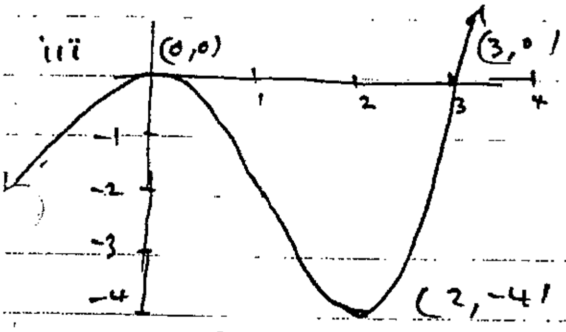
For pt of inflexion $\frac{d^2y}{dx^2} = 0$

$\therefore 6x - 6 = 0$

$x = 1$

pt of inflexion at $(1,-2)$

iii



iv) $x < 0,\ x > 2$

v) $x < 1$

vi) $x^3 - 3x^2 = -kx + 3k$

$x^3 - 3x^2 + kx - 3k = 0$

$x^2(x-3) + k(x-3) = 0$

$(x^2 + k)(x-3) = 0$

$\therefore k > 0$

Question 10

a) $\int_p^{p+1} 3x^2\,dx = \left[x^3\right]_p^{p+1}$

$\therefore (p+1)^3 - p^3 = 19$

$[p+1-p][(p+1)^2 + (p+1)p + p^2] = 19$

$[p^2 + 2p + 1 + p^2 + p + p^2] = 19$

$3p^2 + 3p - 18 = 0$

$3(p^2 + p - 6) = 0$

$3(p+3)(p-2) = 0$

$\therefore p = -3$ or 2

b) i) $t = 0,\ D = 3$

ii) $D = -1$ 1 metre of sandbar sho

iii) when $D = 3$

$3 = 2\cos\frac{4\pi}{25}t + 1$

$\cos\frac{4\pi}{25}t = 1$

$\therefore \frac{4\pi}{25}t = 0, 2\pi, 4\pi \ldots$

$t = 0, 12.5, 25 \ldots$

$\therefore 12\frac{1}{2}$ hours between high tides

iv)